

Exercise 146

For the following exercises, simplify each expression by writing it in terms of sines and cosines, then simplify. The final answer does not have to be in terms of sine and cosine only.

$$\frac{1 + \tan^2 \alpha}{1 + \cot^2 \alpha}$$

Solution

$$\begin{aligned}\frac{1 + \tan^2 \alpha}{1 + \cot^2 \alpha} &= \frac{\sec^2 \alpha}{\csc^2 \alpha} \\ &= \frac{\frac{1}{\cos^2 \alpha}}{\frac{1}{\sin^2 \alpha}} \\ &= \frac{\sin^2 \alpha}{\cos^2 \alpha} \\ &= \tan^2 \alpha\end{aligned}$$